**Mathematical Working Principle of the Median Filter**

**Definition:**

The **median filter** is a **non-linear** digital filtering technique used to **remove noise** from an image or signal while **preserving edges**.

**Mathematical Steps:**

Let the image be represented as a 2D matrix I(x, y), where x and y are the spatial coordinates.

Let the neighborhood (filter window) be of size m×n centered at (x, y).

**Step-by-Step:**

1. **Select a window** of size m×n centered at pixel (x, y).
2. **Extract the pixel values** within this window:

W={I(i,j) ∣ i∈[x−a,x+a], j∈[y−b,y+b]}

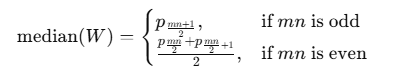
where:

* + a = m/2
  + b = n/2

1. **Sort** the pixel values in WW in ascending order:

Wsorted = {p1,p2,...,pmn}

1. **Compute the median**:



1. **Replace** the center pixel value I(x,y)I(x, y) with the computed median:

I′(x,y)=median(W)I'(x, y) = \text{median}(W)

**Practical Application of Median Filter**

**1. Salt-and-Pepper Noise Removal in Images**

* Salt-and-pepper noise causes random black (0) and white (255) pixels.
* Median filter removes these outliers by replacing them with the **median of their neighborhood**, which is usually a valid, non-noisy value.

📌 *Example:*

Before:

[100, 255, 102]

[ 98, 0, 101]

[ 97, 100, 103]

After Median Filter (3x3):

[100, 100, 102]

[ 98, 100, 101]

[ 97, 100, 103]

**2. Preserving Edges in Medical Imaging (e.g., MRI, X-ray)**

* Unlike averaging filters, which blur edges, the median filter **preserves sharp transitions** while removing noise.
* This makes it useful in medical image pre-processing.

**3. Signal Processing (1D median filter)**

* In ECG or other biosignals, sudden spikes or dropouts (outliers) are removed using median filtering while keeping the overall waveform shape.

**4. Video Surveillance and Traffic Cameras**

* Helps in cleaning up noisy frames where lighting or dust causes pixel flickering.

**🧾 Summary Table**

| **Feature** | **Median Filter** |
| --- | --- |
| Type | Non-linear filter |
| Operation | Replace center value with the median |
| Preserves edges? | Yes |
| Good at removing | Salt-and-pepper noise, impulse noise |
| Common window size | 3×3, 5×5, 7×7 (odd-sized, usually square) |
| Formula for median | See formula above |